Efficient Firing Costs*

Guillaume NÉVO† and Cesar ZAMBRANO‡

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Abstract

ABSTRACT

*We are grateful to...
†Department of Economics, Uppsala University. Email: guillaume.nevo@nek.uu.se
‡Department of Economics, New York University. Email: cesar.zambrano@nyu.edu

Introduction

We investigate the efficiency of firing costs in a macro-labor setting, in terms of total output, and their distributional impact on workers. Employment protection laws are often at the heart of labor economics, and many papers have been looking at their impacts on workers. Surprisingly though, few labor market models have actually studied the effect of firms actively looking to replace their workers. This chapter intends to fill this gap.

Standard search-and-matching models are ill-suited to look at such problem. In the standard DMP model, both workers and firms are assumed to fully commit to their current contract, in which workers provide the firm with a unit of labor in exchange of which they receive a predetermined wage they bargained over, until the match is exogenously destroyed. Allowing for on-the-job search, extends search on the worker sides - workers now partially commit to the contract. If they are matched with a better firm, they simply renege on their current firm. Such behavior on the firm side is ruled out by assumption. The only two ways a match can usually be terminated is through exogenous destruction, or when the match surplus falls below a predetermined value - 0 or some firing cost the firm would have to pay. This asymmetry might be reflect the strict labor protections in some countries in which firing a worker is nearly impossible, but in others it seems like an unrealistic assumption often made for tractability.

One could argue that, as in most search-and-matching models, the notion of firm itself is ill-defined. A worker quitting, and another worker being hired within the same firm could be modeled as a match being destroyed, and a new, independent vacancy being filled. From a data perspective, distinguishing what is a new hire compared to a replacement hire is also an arduous problem. Yet surveys of employers argue that worker's replacement is a large part of their hiring policy (Mercan and Schoefer 2020 argue that employers classify 56% of their vacancies as quit-driven replacement hiring into old jobs). Having a framework to understand the drivers of worker's replacement, after a quit, or a firing is therefore important.

In this project we build on Cahuc, Postel-Vinay, and Robin (2006) (henceforth CPVR). They propose a search-and-matching model in which workers can search on-the-job and rebargain their wage. Importantly, firms have full commitment to the contract. We extend their framework by allowing firms to actively search for new workers too. If a new worker is met by the firm, it can replace its own worker by paying a firing cost, or rebargain the wage of its current worker. In that sense, in our model firms remain more committed to the match than workers. This framework is close in spirit to Kiyotaki and Lagos (2007), in which both workers and firms actively search for better matches, or Acharya and Wee (2018), which reverses the usual commitment by committing workers to their contract, but allowing firms to replace their workers.

The aim of this project is to study the impact of firing costs on total output, unemployment rate, and inequalities. We derive the transition rules and bargaining positions of the different meetings between workers and firms, solve for the surplus equation, and compute numerically the distribution of workers and firms across matches in our extended version of CPVR. Contrary to what proponents of stringer employment protection laws argue, we find that firing costs hurt low-type workers as firms prefer not hiring them rather than being unable to replace them later. On the contrary, higher firing costs increase the bargaining position of high-type workers without reducing their employment level, increasing their wage. Aggregate effects are small though.

Future iteration of this project will look deeper at how the wage distribution varies depending on the level of firing costs, and will investigate the role of severance payments. We also want to look at the effect of counter-cyclical firing costs policy, that might better protect low-type workers in recessions, without hurting them too much in booms.

Policy implications. Many employment protection laws are devised around firing costs for firms. Gaining a better understanding of the implications of them on output, unemployment rate, and the distribution of wages is therefore an important question for policy making. By estimating the model on a country in which these protections are very stringent, such as France, we should be able to quantify how much output is potentially lost, and how higher unemployment rate is because of these high firing costs.

Related literature

A vast theoretical literature on the effects of firing costs on labor market and aggregate outcomes exist, and it would be a daunting task to give an overview of it, nor to do justice to its most important contributions. A first strand of papers focused on the effects of firing costs on labor market flows (Bentolila and Bertola 1990, Garibaldi 1998), discussing their efficiency in reducing layoffs (Chen and Zoega 1999), and the adverse selection issues associated with hiring workers of unknown quality (Kugler and Saint-Paul 2004). These papers looked mainly at the level of unemployment, but pay little attention to the aggregate effect of firing costs. In addition, these papers consider firing as a tool to layoff part of their workforce - not as a mechanism to replace workers.

More recent papers have used directed search frameworks with two-sided limited commitment to show how firing costs can explain the small response of unemployment to productivity shocks documented by Shimer (2005). Rudanko (2009) develops a model of optimal wage contracting with wage rigidity, in which firms can fire workers. In her paper, a worker hired in a boom is paid a hire wage, and the firm has an incentive to fire her once productivity decreases. Menzio and Moen (2010), Snell and Thomas (2011) and Snell, Thomas, and Wang (2015) have firms to commit to a wage policy, but not to an employment policies in the sense that firms can dismiss workers, and potentially replace them by new hires. The difference between these papers and ours is twofold: first, firms in theirs have a mass of workers, and firms decide to layoff some of them, and hire others in the next period. Firms are not actively searching for workers, nor replacing them once they match with a better worker. Second, firms can fire at will, and for free. The focus is in explaining wage rigidity and employment volatility, not to study the impact of firing costs on output and inequalities.

As will be clear below, one key part of the problem we study is the value of a vacant job. A few other papers have taken a similar approach as ours. Once a match is destroyed, either exogenously, or following a worker's transition, the job becomes vacant again instead of disappearing. Cahuc and Postel-Vinay (2002) use such a framework to study the impact of dual labor markets. They compare short-term contract that last one period and end without firing costs and long-term contracts that can only be terminated by paying a fixed cost. Hornstein, Krusell, and Violante (2007) look at irreversible capital investment in a frictional labor-market. Firms invest in capital and match with workers to operate that capital. When a worker leaves, the firm becomes vacant but the capital is not lost. Mercan and Schoefer (2020) studies the role of vacancies in "creating" new vacancies. The cost of posting a new vacancy depends on the stock of new vacancies in the labor market - the more new vacancies, the higher the cost of creating a new one. As workers search on-the-job, more vacancies lead to more meeting, which might lead to more switches, and therefore more vacancies. They call this mechanism a ”vacancy chain”. Although the value of a vacant job is crucial in these papers, none uses it to investigate worker replacement.

Finally, a vast empirical literature on the effects of firing costs also exist (see for instance Kugler 1999, Abowd and Kramarz 2003, Dube, Freeman, and Reich 2010 or Sestito and Viviano 2018, for studies in Colombia, France, California, or Italy). As our paper is mainly theoretical, we will not delve into it.

Outline

This chapter is organized as follows: section 1 describes the general environment we study. Section 2 discusses at length the different meetings between workers and firms, and the resulting transition rules and bargaining outcomes. Section 3 computes the surplus of the match, the key object we want to analyze, and section 4 presents the results of our model using plausible parameters, and discusses the effect of firing costs on the surplus of a match and the distribution of workers across matches.

1 A model of the labor market with firing costs

We study a labor market with two-sided heterogeneity. On one side of the market, a mass 1 of risk-neutral and infinitely-lived workers with fixed productivity x drawn from a distribution $F(\cdot)$. On the other side, an endogenous mass N of risk-neutral firms with fixed productivity y drawn from a distribution $G(\cdot)$, which face a fixed probability of destruction each period δ. When matched, firms and workers produce a single good through $z = x \cdot y$. Time is continuous and we focus on a full information, stationary economy.

Workers can be unemployed or employed. Unemployed and employed workers receive unemployment benefits b and wage w. Similarly, firms can be vacant or occupied. A vacant firm costs c_v per period to maintain, and an occupied firm makes instantaneous profits $z - w$. Wages are determined through Nash-bargaining between workers and firms, and potentially renegotiated whenever there is a credible threat from one side. We denote by α the bargaining power of the worker.

Search is random for all workers and firms, and employed workers and occupied firms have a reduced search efficiency. We come back to the contact rate, labor market tightness and matching function in section 3.1.

Following CPVR, workers search on- and off-the-job. When unemployed workers meet a vacant firm, they bargain over the wage through Nash-bargaining where their respective outside options are to remain unemployed and vacant. When employed workers meet a vacant firm, the current and the new firms compete to retain/poach the worker. As in CPVR, the outcome of the alternating offer game leads the worker to stay/move to the firm with which the surplus of the match is the highest, and to Nash-bargain with that firm. The outside options of the worker and the firm are the surplus from their previous match and the value of remaining vacant.

Building on this logic, we allow firms to search ”on-the-job”. Firms actively look for a worker both when vacant and when occupied. When vacant firms meet an unemployed worker, the situation is the same as when an unemployed worker meets a vacant firm. When a firm employing a worker meets an unemployed worker though, the firm will have to pay a firing cost c_f to replace its worker. The incumbent worker and the new worker will compete for the job. The outcome of this alternating offer game leads the firm to retain or replace its worker depending on which of the current surplus, or the new surplus minus the firing costs is higher. The outside option of the worker that remains or is hired by the firm is unemployment in both cases. The outside option of the firm is the whole surplus it could have extracted by hiring the new worker, ie the surplus of the new match minus the firing costs, if the incumbent worker stays, or the surplus of its current match if the new worker is hired.

Employed workers and occupied firms can also meet one another. This situation is a straightforward extension of the mechanisms described above. The next section runs through the details of the bargaining process, hence we postpone a thorough discussion for now.

Firing costs depend on both the type of the current worker x and the firm y, and a fraction

γ of them are rebated to the fired worker, capturing the fact that severance packages tend to be proportional to a worker's wage. Notice that firing costs are the main asymmetry between workers and firms. Both workers and firms face limited commitment, but the magnitude of firing costs modulate firms' commitment: with zero firing costs, there is perfect symmetry between firms and workers, whereas with infinite firing costs, we are back to the usual CPVR model, in which firms are fully committed.

Matches are exogenously destroyed at rate η, in which case the worker goes back to unemployment, and the job becomes vacant. When firms are destroyed, as mentioned above, the worker goes back to unemployment if that firm was occupied. New vacant firms enter by paying a fixed-cost c_e and draw y from $G(\cdot)$.

Denote by V, value function of employed worker, U of unemployed worker, J of occupied job, K of vacant job. Define the surplus of a match S by: $S = V - U + J - K$. Finally, following Lise and Postel-Vinay (2020), denote by σ the share of the surplus of the match extracted by a worker: $\sigma = \frac{V-U}{S}$.

We describe first the transition rules and the outcomes of the bargaining process when a firm and a worker meet, before turning to the value function and the surplus characterization.

2 Transition rules and bargaining

During a match, or from unemployment or a vacant job, workers and firms are actively looking for a potential new match. In order to write down the value function of unemployed and employed workers, and vacant and occupied jobs, we need to consider all the different meetings that each of them can face. An employed worker, for instance, can meet a vacant job, and an occupied job. In both situations, she can be poached by the new job, renegotiate with her current job, or nothing can happen. We characterize the transition rules and the resulting bargaining for all of those cases in this section.

Denote workers and firms by their type x, y, and a match by the tuple (x, y, σ), and workers and firms that they meet by $\tilde{x}$, or $\tilde{y}$. If those workers or firms are already in their own match, denote it by the tuple $(\tilde{x}, \tilde{y}, \tilde{\sigma})$.

2.1 Unemployed worker x meets vacant job $\tilde{y}$

The simplest case focuses on an unemployed worker meeting a vacant job. Following the convention above, denote them by x for the unemployed worker and $\tilde{y}$ for the new firm. The current value functions are $U(x)$ and $K(\tilde{y})$. Finally denote by σ_{uv} the result of the Nash-bargaining between the unemployed worker and the vacant firm if the match is created. The new value functions, were the match to happen, are $V(x, \tilde{y}, \sigma_{uv})$ and $J(x, \tilde{y}, \sigma_{uv})$, as a match $(x, \tilde{y})$ would be created, with surplus sharing rule σ_{uv}. The match is beneficial for both sides if $V(x, \tilde{y}, \sigma_{uv}) - U(x) > 0$ and $J(x, \tilde{y}, \sigma_{uv}) - K(\tilde{y}) > 0$, otherwise the worker stays unemployed and the job vacant.

Turning to the Nash-bargaining: gain from match is $V(x, \tilde{y}, \sigma_{uv}) - U(x)$ for worker and $J(x, \tilde{y}, \sigma_{uv}) - K(\tilde{y})$ for job, whereas the outside options are 0 for both. The Nash-bargaining problem can be written as:

$$\text{argmax}_w \left(J(x, \tilde{y}, \sigma_{uv}) - K(\tilde{y})\right)^{1-\alpha} \left(V(x, \tilde{y}, \sigma_{uv}) - U(x)\right)^{\alpha}$$

Taking the first order conditions:

$$\begin{aligned}(1-\alpha)\frac{\partial J}{\partial w} &\left(J(x, \tilde{y}, \sigma_{uv}) - K(\tilde{y})\right)^{-\alpha} \left(V(x, \tilde{y}, \sigma_{uv}) - U(x)\right)^{\alpha} \\ + \alpha\frac{\partial V}{\partial w} &\left(J(x, \tilde{y}, \sigma_{uv}) - K(\tilde{y})\right)^{1-\alpha} \left(V(x, \tilde{y}, \sigma_{uv}) - U(x)\right)^{\alpha-1} = 0\end{aligned}$$

As $\frac{\partial J}{\partial w} = -\frac{\partial V}{\partial w}$, we get:

$$(1-\alpha)\left(V(x, \tilde{y}, \sigma_{uv}) - U(x)\right) = \alpha\left(J(x, \tilde{y}, \sigma_{uv}) - K(\tilde{y})\right)$$

Plugging S, we have:

$$V(x, \tilde{y}, \sigma_{uv}) - U(x) = \alpha S(x, \tilde{y}, \sigma_{uv})$$

Hence the surplus splitting rule follows: $\sigma_{uv} = \alpha$

From the firm's side:

$$J(x, \tilde{y}, \sigma_{uv}) - K(\tilde{y}) = (1 - \alpha) S(x, \tilde{y}, \sigma_{uv})$$

To summuarize the situation in which an unemployed worker meets a vacant job:

- Match is created if $S(x, \tilde{y}, \sigma_{uv}) > 0$;
- $V(x, \tilde{y}, \sigma_{uv}) - U(x) = \sigma_{uv} S(x, \tilde{y}, \sigma_{uv})$;
- $J(x, \tilde{y}, \sigma_{uv}) - K(\tilde{y}) = (1 - \sigma_{uv}) S(x, \tilde{y}, \sigma_{uv})$;
- $\sigma_{uv} = \alpha$.

And the gains for the unemployed value function that we will use later when characterizing the surplus:

$$V(x, \tilde{y}, \sigma_{uv}) - U(x) = \alpha S(x, \tilde{y}, \sigma_{uv})$$

As expected, we recover the usual bargaining results between an unemployed worker and a firm. The notation we developed here allows us to also characterize the more interesting cases in which, for instance, an occupied job meets a worker, employed or not, and weights whether to pay the firing costs to poach the new worker, or renegotiate with its current worker to extract a larger share of the surplus. We repeat this exercise for all the other possible cases. The structure of the different cases is identical. Once the Nash-bargaining problem is defined, we can take first order conditions, and recover the conditions for the match to be created, the new value functions and bargaining positions, and the gains and losses of each value functions. We skip the unnecessary algebra.

2.2 Unemployed worker x meets occupied job $(\tilde{x}, \tilde{y}, \tilde{\sigma})$

We turn to the case in which an unemployed worker of type x meets occupied job of firm quality $\tilde{y}$ currently matched with worker $\tilde{x}$, in which the worker extracts a share of the surplus $\tilde{\sigma}$, ie $(\tilde{x}, \tilde{y}, \tilde{\sigma})$. The unemployed worker can either take the job, and therefore the firm replaces its current worker, or the worker remains unemployed. Following the bargaining protocol in CPVR, both workers compete for the job. If the firm replaces its worker, it manages to extract all the surplus from its current match, minus the firing costs. Hence, the new worker takes job if $V(x, \tilde{y}, \sigma_{uo}) - U(x) > 0$ and $J(x, \tilde{y}, \sigma_{uo}) - K(\tilde{y}) - c_f > S(\tilde{x}, \tilde{y}, 0)$, otherwise the worker stays unemployed and the job occupied by current worker. σ_{uo} denotes the share of the new surplus the worker extracts in the new match.

The Nash-bargaining is described as follow: the gains from match are $V(x, \tilde{y}, \sigma_{uo}) - U(x)$ for the worker and $J(x, \tilde{y}, \sigma_{uo}) - K(\tilde{y}) - c_f$ for the firm, whereas the outside option of the worker is 0, and of the job is $S(\tilde{x}, \tilde{y}, 0)$. Notice the last argument of the surplus here in the outside option of the firm - as the firm extracts all the surplus of its previous match, the share left to the worker falls to 0.

Solving the Nash-bargaining problem, we obtain:

- Match is replaced if $S(x, \tilde{y}, \sigma_{uo}) > c_f + S(\tilde{x}, \tilde{y}, 0)$;
- $V(x, \tilde{y}, \sigma_{uo}) - U(x) = \sigma_{uo} S(x, \tilde{y}, \sigma_{uo})$;
- $J(x, \tilde{y}, \sigma_{uo}) - K(\tilde{y}) = (1 - \sigma_{uo}) S(x, \tilde{y}, \sigma_{uo})$;
- $\sigma_{uo} = \alpha \frac{S(x, \tilde{y}, \sigma_{uo}) - c_f - S(\tilde{x}, \tilde{y}, 0)}{S(x, \tilde{y}, \sigma_{uo})}$.

And the gains for the unemployed value function are:

$$V(x, \tilde{y}, \sigma_{uo}) - U(x) = \alpha S(x, \tilde{y}, \sigma_{uo}) - \alpha c_f - \alpha S(\tilde{x}, \tilde{y}, 0)$$

2.3 Vacant job y meets unemployed worker $\tilde{x}$

This case focuses on a vacant job of type y meeting an unemployed worker of type $\tilde{x}$. It is exactly identical to the one in which unemployed worker of type x meets a vacant job of type $\tilde{y}$ described above. To summarize:

- Match is created if $S(\tilde{x}, y, \sigma_{vu}) > 0$;
- $V(\tilde{x}, y, \sigma_{vu}) - U(\tilde{x}) = \sigma_{vu} S(\tilde{x}, y, \sigma_{vu})$;
- $J(\tilde{x}, y, \sigma_{vu}) - K(y) = (1 - \sigma_{vu}) S(\tilde{x}, y, \sigma_{vu})$;
- $\sigma_{vu} = \alpha$.

And the gains for the vacant job value function are:

$$J(\tilde{x}, y, \sigma_{vu}) - K(y) = (1 - \alpha) S(\tilde{x}, y, \sigma_{vu})$$

2.4 Vacant job y meets employed worker $(\tilde{x}, \tilde{y}, \tilde{\sigma})$

The case in which a vacant job of type y meets an employed worker of type $\tilde{x}$ currently in a match characterized by the tuplet $(\tilde{x}, \tilde{y}, \tilde{\sigma})$ is identical to the poaching of an employed worker in (x, y, σ) by a vacant job $\tilde{y}$, described in the next subsection. We postpone the discussion, and only summarize here the results.

The worker is poached by the vacant job if $S(\tilde{x}, y, 1) > V(\tilde{x}, y, \sigma_{ve}) - U(\tilde{x}) > S(\tilde{x}, \tilde{y}, 1) > V(\tilde{x}, \tilde{y}, \tilde{\sigma}) - U(\tilde{x})$ and $J(\tilde{x}, y, \sigma_{ve}) - K(y) > 0$, otherwise the worker stays at her current firm and the job remains vacant. The results of the bargaining are:

- Worker is poached if $S(\tilde{x}, y, \sigma_{ve}) > S(\tilde{x}, \tilde{y}, 1)$;
- $V(\tilde{x}, y, \sigma_{ve}) - U(\tilde{x}) = \sigma_{ve} S(\tilde{x}, y, \sigma_{ve})$;
- $J(\tilde{x}, y, \sigma_{ve}) - K(y) = (1 - \sigma_{ve}) S(\tilde{x}, y, \sigma_{ve})$;

- $\sigma_{ve} = \frac{S(\tilde{x},\tilde{y},1)+\alpha(S(\tilde{x},y,\sigma_{ve})-S(\tilde{x},\tilde{y},1))}{S(\tilde{x},y,\sigma_{ve})}$.

And the gains in terms of the vacant job's value function are:

$$J(\tilde{x}, y, \sigma_{ve}) - K(y) = (1 - \alpha)(S(\tilde{x}, y, \sigma_{ve}) - S(\tilde{x}, \tilde{y}, 1))$$

2.5 Employed worker (x, y, σ) meets vacant job $\tilde{y}$

The case in which an employed worker of type x in her current job (x, y, σ), meets a vacant job of type $\tilde{y}$ follows directly CPVR. The worker can either be poached by her new firm, renegotiate with her current firm, or nothing happens, depending on the potential value of the new surplus, and the previous surplus. We describe each case alternatively.

2.5.1 Poaching

Both firms will compete to poach or retain the worker. If the new match is more productive, the new firm will outbid the current firm. The worker will be able to extract all the surplus from her current match, and bargain over a share of the extra surplus created by the new match. Hence, the worker in (x, y, σ) moves to the new job if $S(x, \tilde{y}, \sigma_{ev}) > V(x, \tilde{y}, \sigma_{ev}) - U(x) > S(x, y, 1) > V(x, y, \sigma) - U(x)$ and $J(x, \tilde{y}, \sigma_{ev}) - K(\tilde{y}) > 0$, with σ_{ev} the share of the new surplus the worker will collect. The Nash-bargaining is characterized by the gains from match for the worker, $V(x, \tilde{y}, \sigma_{ev}) - U(x)$, for the new firm, $J(x, \tilde{y}, \sigma_{ev}) - K(\tilde{y})$, and the outside options $S(x, y, 1)$, and 0.

Solving the Nash-bargaining, we obtain that:

- Worker moves if $S(x, \tilde{y}, \sigma_{ev}) > S(x, y, 1)$;
- $V(x, \tilde{y}, \sigma_{ev}) - U(x) = \sigma_{ev} S(x, \tilde{y}, \sigma_{ev})$;
- $J(x, \tilde{y}, \sigma_{ev}) - K(\tilde{y}) = (1 - \sigma_{ev}) S(x, \tilde{y}, \sigma_{ev})$;
- $\sigma_{ev} = \frac{S(x,y,1)+\alpha(S(x,\tilde{y},\sigma_{ev})-S(x,y,1))}{S(x,\tilde{y},\sigma_{ev})}$.

And the gains for the employed's value function are:

$$V(x, \tilde{y}, \sigma_{ev}) - V(x, y, \sigma) = S(x, y, 1) + \alpha\left(S(x, \tilde{y}, \sigma_{ev}) - S(x, y, 1)\right) - \sigma S(x, y, \sigma)$$

Whereas the losses for the firm's value function are:

$$K(y) - J(x, y, \sigma) = -(1 - \sigma)S(x, y, \sigma)$$

2.5.2 Renegotiation

If the new surplus is not sufficiently high for the worker to be poached, but is higher than the worker's current bargaining position, a renegotiation happens. The worker extracts all surplus her prospective match would give her, and bargains over what is left of the surplus. Mathematically, the worker in (x, y, σ) stays at her job but renegotiates her wage up if $S(x, y, 1) > V(x, y, \sigma_{rv}) - U(x) > S(x, \tilde{y}, 1) > V(x, y, \sigma) - U(x)$ and $J(x, y, \sigma_{rv}) - K(y) > 0$, with σ_{rv} the new surplus share the worker collects. The Nash-bargaining are such that the gains from the match are $V(x, y, \sigma_{rv}) - U(x)$ for worker and $J(x, y, \sigma_{rv}) - K(y)$ for the firm, whereas the outside options are $S(x, \tilde{y}, 1)$, and 0. Solving it gives us:

- Worker stays and renegotiates if $S(x, y, \sigma_{rv}) > S(x, \tilde{y}, 1) > \sigma S(x, y, \sigma)$;
- $V(x, y, \sigma_{rv}) - U(x) = \sigma_{rv} S(x, y, \sigma_{rv})$;
- $J(x, y, \sigma_{rv}) - K(y) = (1 - \sigma_{rv})S(x, y, \sigma_{rv})$;
- $\sigma_{rv} = \frac{S(x,\tilde{y},1)+\alpha(S(x,y,\sigma_{rv})-S(x,\tilde{y},1))}{S(x,y,\sigma_{rv})}$.

The gains from the renegotiation for the worker are:

$$V(x, y, \sigma_{rv}) - V(x, y, \sigma) = S(x, \tilde{y}, 1) + \alpha\left(S(x, y, \sigma_{rv}) - S(x, \tilde{y}, 1)\right) - \sigma S(x, y, \sigma)$$

Whereas the losses for the firm are:

$$J(x,y,\sigma_{rv}) - J(x,y,\sigma) = (1-\alpha)\left(S(x,y,\sigma_{rv}) - S(x,\tilde{y},1)\right) - (1-\sigma)S(x,y,\sigma)$$

And the change in surplus are:

$$V(x,y,\sigma_{rv}) - V(x,y,\sigma) + J(x,y,\sigma_{rv}) - J(x,y,\sigma) = S(x,y,\sigma_{rv}) - S(x,y,\sigma)$$

2.6 Employed worker (x,y,σ) meets occupied job $(\tilde{x},\tilde{y},\tilde{\sigma})$

We turn to the case in which an employed worker of type x in (x,y,σ) meets occupied job of type $\tilde{y}$ in $(\tilde{x},\tilde{y},\tilde{\sigma})$. Here, were the firm willing to replace its worker, it would have to pay the firing costs. Depending on how high the new surplus net of the firing costs is, the worker decides to move to the new job, to renegotiate with her current firm, or nothing happens.

2.6.1 Poaching

Now both firms compete for the new worker, and both workers, matched at each firm, compete for the new job. The most the current firm can offer to its worker to keep her is $S(x,y,1)$. Similarly, the most worker $\tilde{x}$ can offer to firm $\tilde{y}$ not to be fired is $S(\tilde{x},\tilde{y},0)$. The new firm would have to pay the firing costs, and would receive $J(x,\tilde{y},\sigma_{eo}) - K(\tilde{y}) - c_f$ in the new match, when the worker would receive $V(x,\tilde{y},\sigma_{eo}) - U(x)$. Thus, the worker in (x,y,σ) moves to the new job if $V(x,\tilde{y},\sigma_{eo}) - U(x) > S(x,y,1) > V(x,y,\sigma) - U(x)$ and $J(x,\tilde{y},\sigma_{eo}) - K(\tilde{y}) - c_f > S(\tilde{x},\tilde{y},0) > J(\tilde{x},\tilde{y},\tilde{\sigma}) - K(\tilde{y})$. In that case, the worker and the firm extract all the surplus from their previous matches and bargain over a share of the new extra surplus.

The Nash-bargaining splits the gains from the match, $V(x,\tilde{y},\sigma_{eo}) - U(x)$, for the worker, and, $J(x,\tilde{y},\sigma_{eo}) - K(\tilde{y}) - c_f$, for the firm, versus their outside options $S(x,y,1)$, and $S(\tilde{x},\tilde{y},0)$. Solving it gives us:

- Worker moves to new job if $S(x,\tilde{y},\sigma_{eo}) > S(x,y,1) + c_f + S(\tilde{x},\tilde{y},0)$;
- $V(x,\tilde{y},\sigma_{eo}) - U(x) = \sigma_{eo} S(x,\tilde{y},\sigma_{eo})$;
- $J(x,\tilde{y},\sigma_{eo}) - K(\tilde{y}) = (1-\sigma_{eo}) S(x,\tilde{y},\sigma_{eo})$;
- $\sigma_{eo} = \frac{S(x,y,1)+\alpha\left(S(x,\tilde{y},\sigma_{eo})-S(x,y,1)-c_f-S(\tilde{x},\tilde{y},0)\right)}{S(x,\tilde{y},\sigma_{eo})}$.

The gains for employed's value function are:

$$V(x,\tilde{y},\sigma_{eo}) - V(x,y,\sigma) = S(x,y,1) + \alpha\left(S(x,\tilde{y},\sigma_{eo}) - S(x,y,1) - c_f - S(\tilde{x},\tilde{y},0)\right) - \sigma S(x,y,\sigma)$$

And the losses for firm's value function are:

$$K(y) - J(x,y,\sigma) = -(1-\sigma) S(x,y,\sigma)$$

2.6.2 Renegotiation

If the new surplus is not high enough compared to her current match, the worker can still use this potential poaching to renegotiate her wage up. For a renegotiation to take place, the new firm worker x is meeting has to be a credible threat. Firm $\tilde{y}$ uses the meeting with worker x to bargain up its position with its current worker $\tilde{x}$. Both workers compete for the job at $\tilde{y}$. For the firm to be willing to replace its worker, it must be that the gains from the match with x, net of the firing costs, are higher than the surplus with $\tilde{x}$ - otherwise worker $\tilde{x}$ is a better match for firm $\tilde{y}$ and it would not be willing to replace $\tilde{x}$ by x.

If firm $\tilde{y}$ is willing to replace its worker by x, both firms now compete for x. If the worker remains at y, it must be that the value for the worker to remain at y is higher than what she could have extracted from $\tilde{y}$ by moving - what is left once firm $\tilde{y}$ has extracted the whole surplus of its match with $\tilde{x}$. Thus the outside option of worker x in this case is $S(x,\tilde{y},\sigma_{reneg}) - c_f - S(\tilde{x},\tilde{y},0)$, with $\sigma_{reneg} = \frac{S(x,\tilde{y},\sigma_{reneg})-c_f-S(\tilde{x},\tilde{y},0)}{S(x,\tilde{y},\sigma_{reneg})}$. A worker in (x,y,σ) therefore stays but renegotiates if

$S(x,y,1) > V(x,y,\sigma_{ro}) - U(x) > S(x,\tilde{y},\sigma_{reneg}) - c_f - S(\tilde{x},\tilde{y},0) > V(x,y,\sigma) - U(x)$ and $J(x,y,\sigma_{ro}) - K(y) > 0$.

The Nash-bargaining is then: gains from the match are $V(x,y,\sigma_{ro}) - U(x)$ and $J(x,y,\sigma_{ro}) - K(y)$, and the outside options are $S(x,\tilde{y},\sigma_{reneg}) - c_f - S(\tilde{x},\tilde{y},0)$, and 0. Solving for the bargaining give us:

- Worker stays and renegotiates if $S(x,y,\sigma_{ro}) > S(x,\tilde{y},\sigma_{reneg}) - c_f - S(\tilde{x},\tilde{y},0) > \sigma S(x,y,\sigma)$;
- $V(x,y,\sigma_{ro}) - U(x) = \sigma_{ro} S(x,y,\sigma_{ro})$;
- $J(x,y,\sigma_{ro}) - K(y) = (1-\sigma_{ro}) S(x,y,\sigma_{ro})$;
- $\sigma_{ro} = \frac{S(x,\tilde{y},\sigma_{reneg}) - c_f - S(\tilde{x},\tilde{y},0) + \alpha\left(S(x,y,\sigma_{ro}) - S(x,\tilde{y},\sigma_{reneg}) + c_f + S(\tilde{x},\tilde{y},0)\right)}{S(x,y,\sigma_{ro})}$;
- $\sigma_{reneg} = \frac{S(x,\tilde{y},\sigma_{reneg}) - c_f - S(\tilde{x},\tilde{y},0)}{S(x,\tilde{y},\sigma_{reneg})}$.

The gains from renegotiation for the worker are:

$$\begin{aligned} V(x,y,\sigma_{ro}) - V(x,y,\sigma) = {} & S(x,\tilde{y},\sigma_{reneg}) - c_f - S(\tilde{x},\tilde{y},0) \\ & + \alpha\left(S(x,y,\sigma_{ro}) - S(x,\tilde{y},\sigma_{reneg}) + c_f + S(\tilde{x},\tilde{y},0)\right) - \sigma S(x,y,\sigma) \end{aligned}$$

Whereas the losses for the firm are:

$$J(x,y,\sigma_{ro}) - J(x,y,\sigma) = (1-\alpha)\left(S(x,y,\sigma_{ro}) - S(x,\tilde{y},\sigma_{reneg}) + c_f + S(\tilde{x},\tilde{y},0)\right) - (1-\sigma) S(x,y,\sigma)$$

The total change in surplus is then:

$$V(x,y,\sigma_{ro}) - V(x,y,\sigma) + J(x,y,\sigma_{ro}) - J(x,y,\sigma) = S(x,y,\sigma_{ro}) - S(x,y,\sigma)$$

2.7 Occupied job (x, y, σ) meets unemployed worker $\tilde{x}$

The former to last case looks at the meeting between of an occupied job (x, y, σ) with an unemployed worker $\tilde{x}$. Depending on the surplus of the potential new match, firm y can decide to replace its current worker x by worker $\tilde{x}$ and pay the firing costs, renegotiate with its current worker to extract a larger share of the surplus, or nothing can happen.

2.7.1 Replacing

Replacing worker x by an unemployed worker $\tilde{x}$ is symmetric to the case in which an unemployed worker meets an occupied job and is hired, that we discussed above. We simply summarize here the outcome of the bargaining: a firm (x, y, σ) replaces its worker by an unemployed worker $\tilde{x}$ if $S(\tilde{x}, y, \sigma_{ou}) > J(\tilde{x}, y, \sigma_{ou}) - c_f - K(y) > S(x, y, 0) > J(x, y, \sigma) - K(y)$ and $V(\tilde{x}, y, \sigma_{ou}) - U(\tilde{x}) > 0$. Solving for the bargaining problem we have that:

- Firm replaces its worker if $S(\tilde{x}, y, \sigma_{ou}) > c_f + S(x, y, 0)$;
- $V(\tilde{x}, y, \sigma_{ou}) - U(\tilde{x}) = \sigma_{ou} S(\tilde{x}, y, \sigma_{ou})$;
- $J(\tilde{x}, y, \sigma_{ou}) - K(y) = (1 - \sigma_{ou}) S(\tilde{x}, y, \sigma_{ou})$;
- $\sigma_{ou} = \alpha \frac{S(\tilde{x}, y, \sigma_{ou}) - c_f - S(x, y, 0)}{S(\tilde{x}, y, \sigma_{ou})}$.

The gains for the firm's value function are:

$$J(\tilde{x}, y, \sigma_{ou}) - c_f - J(x, y, \sigma) = (1 - \alpha) \left(S(\tilde{x}, y, \sigma_{ou}) - c_f \right) + \alpha S(x, y, 0) - (1 - \sigma) S(x, y, \sigma)$$

Whereas the losses for worker's value function are:

$$U(x) - V(x, y, \sigma) = -\sigma S(x, y, \sigma)$$

2.7.2 Renegotiation

Similar to previous cases, if the prospective surplus net of the firing costs is not high enough for the firm to be willing to replace its worker, but is still higher than the firm's current bargaining position, a renegotiation happens as the firm has a credible threat to replace its worker otherwise. Both workers will compete for the job. The most worker $\tilde{x}$ can offer is $S(\tilde{x}, y, 0) - c_f$. Firm y will extract all of that surplus and bargain with its current worker over a share of the extra surplus. Hence, firm y renegotiates with its current worker if $S(x, y, 0) > J(x, y, \sigma_{ru}) - K(y) > S(\tilde{x}, y, 0) - c_f > J(x, y, \sigma) - K(y)$, and $V(x, y, \sigma_{ru}) - U(x) > 0$.

The Nash-bargaining can be written as follow: the gains from the match are $V(x, y, \sigma_{ru}) - U(x)$ for the worker and $J(x, y, \sigma_{ru}) - K(y)$ for the firm, whereas their outside options are 0 and $S(\tilde{x}, y, 0) - c_f$. Solving it gives us:

- Worker stays but is forced to renegotiate if $S(x, y, \sigma_{ru}) > S(\tilde{x}, y, 0) - c_f > (1-\sigma)S(x, y, \sigma)$;
- $V(x, y, \sigma_{ru}) - U(x) = \sigma_{ru} S(x, y, \sigma_{ru})$;
- $J(x, y, \sigma_{ru}) - K(y) = (1 - \sigma_{ru}) S(x, y, \sigma_{ru})$;
- $\sigma_{ru} = \alpha \frac{S(x,y,\sigma_{ru}) + c_f - S(\tilde{x},y,0)}{S(x,y,\sigma_{ru})}$.

The gains from renegotiation for the firm are:

$$J(x, y, \sigma_{ru}) - J(x, y, \sigma) = \alpha(S(\tilde{x}, y, 0) - c_f) + (1 - \alpha)S(x, y, \sigma_{ru}) - (1 - \sigma)S(x, y, \sigma)$$

And the losses for the worker are:

$$V(x, y, \sigma_{ru}) - V(x, y, \sigma) = \alpha S(x, y, \sigma_{ru}) + \alpha c_f - \alpha S(\tilde{x}, y, 0) - \sigma S(x, y, \sigma)$$

So that the change in the surplus is:

$$V(x,y,\sigma_{ru}) - V(x,y,\sigma) + J(x,y,\sigma_{ru}) - J(x,y,\sigma) = S(x,y,\sigma_{ru}) - S(x,y,\sigma)$$

2.8 Occupied job (x, y, σ) meets employed worker $(\tilde{x}, \tilde{y}, \tilde{\sigma})$

Finally, the last case looks at the meeting between of an occupied job (x, y, σ) with an employed worker $(\tilde{x}, \tilde{y}, \tilde{\sigma})$. Depending on the surplus of the potential new match, firm y can decide to replace its current worker x by worker $\tilde{x}$ and pay the firing costs, renegotiate with its current worker to extract a larger share of the surplus, or nothing can happen.

2.8.1 Replacing

Replacing its current worker is again the symmetric case of the employed worker (x, y, σ) meeting an occupied job $(\tilde{x}, \tilde{y}, \tilde{\sigma})$. Both workers will compete for job y, whereas both firms compete for worker $\tilde{x}$. Firm y's outside option when bargaining with worker $\tilde{x}$ is to extract all the surplus from its current match, $S(x, y, 0)$. Worker $\tilde{x}$'s outside option is, similarly, $S(\tilde{x}, \tilde{y}, 1)$. Worker $\tilde{x}$ and firm y then bargain over the surplus leftover. Hence, worker x will be replaced if $V(\tilde{x}, y, \sigma_{oe}) - U(\tilde{x}) > S(\tilde{x}, \tilde{y}, 1) > V(\tilde{x}, \tilde{y}, \tilde{\sigma}) - U(\tilde{x})$, and $J(\tilde{x}, y, \sigma_{oe}) - c_f - K(y) > S(x, y, 0) > J(x, y, \sigma) - K(y)$. The new match is created provided that both worker $\tilde{x}$ and firm y are better off in the new match, once firing costs to replace worker x are factored in by firm y.

The Nash-bargaining is then: gains from match are $V(\tilde{x}, y, \sigma_{oe}) - U(\tilde{x})$ and $J(\tilde{x}, y, \sigma_{oe}) - c_f - K(y)$, and the outside options are $S(\tilde{x}, \tilde{y}, 1)$, and $S(x, y, 0)$. Solving it gives us:

- Firm replaces its worker if $S(\tilde{x}, y, \sigma_{oe}) > c_f + S(x, y, 0) + S(\tilde{x}, \tilde{y}, 1)$;
- $V(\tilde{x}, y, \sigma_{oe}) - U(\tilde{x}) = \sigma_{oe} S(\tilde{x}, y, \sigma_{oe})$;
- $J(\tilde{x}, y, \sigma_{oe}) - K(y) = (1 - \sigma_{oe}) S(\tilde{x}, y, \sigma_{oe})$;
- $\sigma_{oe} = \frac{S(\tilde{x},\tilde{y},1) + \alpha\big(S(\tilde{x},y,\sigma_{oe}) - c_f - S(x,y,0) - S(\tilde{x},\tilde{y},1)\big)}{S(\tilde{x},y,\sigma_{oe})}$.

The gains for the firm's value function are:

$$J(\tilde{x}, y, \sigma_{oe}) - c_f - J(x, y, \sigma)$$
$$= (1-\alpha)\left(S(\tilde{x}, y, \sigma_{oe}) - c_f - S(\tilde{x}, \tilde{y}, 1)\right) + \alpha S(x, y, 0) - (1-\sigma)S(x, y, \sigma)$$

Whereas the losses for worker's value function are:

$$U(x) - V(x, y, \sigma) = -\sigma S(x, y, \sigma)$$

2.8.2 Renegotiation

Lastly, mimicking the renegotiation between an employed worker and an occupied job described above - if the prospective surplus net of the firing costs is not high enough, firm y could still have a credible threat against its current worker x and force a renegotiation. Both workers compete for job y and both firms for worker $\tilde{x}$. If worker $\tilde{x}$ were to stay at her current firm $\tilde{y}$, she could extract up to $S(\tilde{x}, \tilde{y}, 1)$. Firm y needs to offer that amount to make worker $\tilde{x}$ indifferent between staying and moving, and has to bear the firing costs to replace its current worker in addition. Firm y would be left with $S(\tilde{x}, y, \sigma_{reneg}) - c_f - S(\tilde{x}, \tilde{y}, 1)$, where $\sigma_{reneg} = \frac{c_f + S(\tilde{x}, \tilde{y}, 1)}{S(\tilde{x}, y, \sigma_{reneg})}$. Thus a firm in (x, y, σ) renegotiates if $S(x, y, 0) > J(x, y, \sigma_{re}) - K(y) > S(\tilde{x}, y, \sigma_{reneg}) - c_f - S(\tilde{x}, \tilde{y}, 1) > J(x, y, \sigma) - K(y)$ and $V(x, y, \sigma_{re}) - U(x) > 0$.

The Nash-bargaining then is: the gains from match are $V(x, y, \sigma_{re}) - U(x)$ and $J(x, y, \sigma_{re}) - K(y)$, whereas the outside options are 0, and $S(\tilde{x}, y, \sigma_{reneg}) - c_f - S(\tilde{x}, \tilde{y}, 1)$. Solving it we get:

- Worker stays but is forced to renegotiate if $S(x, y, \sigma_{re}) > S(\tilde{x}, y, \sigma_{reneg}) - c_f - S(\tilde{x}, \tilde{y}, 1) > (1-\sigma)S(x, y, \sigma)$;
- $V(x, y, \sigma_{re}) - U(x) = \sigma_{re} S(x, y, \sigma_{re})$;
- $J(x, y, \sigma_{re}) - K(y) = (1-\sigma_{re})S(x, y, \sigma_{re})$;
- $\sigma_{re} = \alpha \frac{S(x, y, \sigma_{re}) + c_f - S(\tilde{x}, y, \sigma_{reneg}) + S(\tilde{x}, \tilde{y}, 1)}{S(x, y, \sigma_{re})}$;

- $\sigma_{reneg} = \frac{c_f + S(\tilde{x},\tilde{y},1)}{S(\tilde{x},y,\sigma_{reneg})}$.

Finally the gains and losses from renegotiation are:

$$V(x,y,\sigma_{re}) - V(x,y,\sigma) = \alpha S(x,y,\sigma_{re}) + \alpha c_f - \alpha S(\tilde{x},y,\sigma_{reneg}) + \alpha S(\tilde{x},\tilde{y},1) - \sigma S(x,y,\sigma)$$

$$J(x,y,\sigma_{re}) - J(x,y,\sigma) = \alpha(S(\tilde{x},y,\sigma_{reneg}) - c_f - S(\tilde{x},\tilde{y},1)) + (1-\alpha)S(x,y,\sigma_{re}) - (1-\sigma)S(x,y,\sigma)$$

So that the change in surplus is:

$$V(x,y,\sigma_{re}) - V(x,y,\sigma) + J(x,y,\sigma_{re}) - J(x,y,\sigma) = S(x,y,\sigma_{re}) - S(x,y,\sigma)$$

3 Derivation of the surplus of a match

Having explored the different meeting possibilities between workers and firms, we can turn to writing the value functions of unemployed and employed workers, vacant and occupied jobs, and the Bellman equation for the surplus of a match. We first need to define the contact rates and, in passing, the distribution of workers and firms across the different states.

3.1 Contact rates

Denote by $H(x,y,\sigma)$ the cumulative distribution function (CDF) of the mass of matches between types less than x and less than y, for which the share of the surplus extracted by the worker is less than σ. The associated probability density function (PDF) is $h(x,y,\sigma)$. Denote by $H_u(x)$ the CDF of the mass of unemployed workers of type less than x, and by $H_v(y)$ the one of vacant jobs of type less than y, with PDF $h_u(x)$ and $h_v(y)$.

Denote by u the unemployment rate, and $e = 1-u$ the employment rate, which is also the mass of occupied jobs. Finally denote by $v = N-e$ the mass of vacant jobs, where N is the equilibrium mass of firms, vacant and occupied. We then need to define the total search intensity for workers and firms. We normalize the search intensity of unemployed workers and of vacant firms to 1, and we assume that employed workers and occupied jobs search with a reduced intensity ξ and χ. The total search intensities, given the mass of workers and jobs, are then $s = u + \xi \cdot e$ and $\nu = (N-e) + \chi \cdot e$. By definition, we have: $\int_x \int_y \int_\sigma h(x,y,\sigma)\mathrm{d}\sigma\mathrm{d}y\mathrm{d}x = e$, $\int_x h_u(x)\mathrm{d}x = u$ and $\int_y h_v(y)\mathrm{d}y = v$. Abusing notation, we will often use $h(x,y)$ for $h(x,y) = \int_\sigma h(x,y,\sigma)\mathrm{d}\sigma$ and $H(x,y)$ for $H(x,y) = \int_\sigma H(x,y,\sigma)\mathrm{d}\sigma$.

The tightness of the labor market is given by $\theta = \frac{\nu}{s}$. We assume unemployed workers meet a job with probability $p(\theta)$, and employed workers with probability $\xi p(\theta)$. When meeting a job, the job is empty with probability $\frac{1}{\nu}$, or occupied with probability $\frac{\chi}{\nu}$. Similarly, vacant jobs meet workers with probability $q(\theta)$, and occupied jobs with probability $\chi q(\theta)$. When meeting a worker, the worker is unemployed with probability $\frac{1}{s}$ and employed with probability $\frac{\xi}{s}$.

3.2 Value functions

An unemployed worker of type x collects unemployment benefits b, and searches for a job. The rate of arrival of jobs is $p(\theta)$. With probability $\frac{1}{\nu}$ ($\frac{\chi}{\nu}$) the job is vacant (occupied). From the transition rules and bargaining results above, an unemployed worker accepts a vacant job $\tilde{y}$ if $V(x,\tilde{y},\sigma_{uv}) - U(x) > 0$ and $J(x,\tilde{y},\sigma_{uv}) - K(\tilde{y}) > 0$, and an occupied job $(\tilde{x},\tilde{y},\tilde{\sigma})$ if $V(x,\tilde{y},\sigma_{uo}) - U(x)$ and $J(x,\tilde{y},\sigma_{uo}) - K(\tilde{y}) - c_f > S(\tilde{x},\tilde{y},0)$. She then receives $V(x,\tilde{y},\sigma_{uv}) - U(x)$ or $V(x,\tilde{y},\sigma_{up}) - U(x)$. The value function of an unemployed worker x is then:

$$
\begin{aligned}
&\rho U(x) = b \\
&+ p(\theta)\frac{1}{\nu}\int_{\tilde{y}} \left(V(x,\tilde{y},\sigma_{uv}) - U(x)\right) \mathbb{1}_{\{V(x,\tilde{y},\sigma_{uv})-U(x)>0\}} \mathbb{1}_{\{J(x,\tilde{y},\sigma_{uv})-K(\tilde{y})>0\}} \mathrm{d}H_v(\tilde{y}) \\
&+ p(\theta)\frac{\chi}{\nu}\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{\sigma}} \left(V(x,\tilde{y},\sigma_{uo}) - U(x)\right) \mathbb{1}_{\{V(x,\tilde{y},\sigma_{uo})-U(x)>0\}} \mathbb{1}_{\{J(x,\tilde{y},\sigma_{uo})-K(\tilde{y})-c_f>S(\tilde{x},\tilde{y},0)\}} \mathrm{d}H(\tilde{x},\tilde{y},\tilde{\sigma})
\end{aligned}
$$

The value function of a vacant job y is very similar. Instead of receiving unemployment benefits, the firm has to pay a cost c_v to maintain the vacancy open. In addition, the firm has a risk of being exogenously destroyed with probability δ. The continuation value in that case is 0. The value function then reads:

$$
\begin{aligned}
&\rho K(y) = -c_v + \delta(0 - K(y)) \\
&+ q(\theta)\frac{1}{s}\int_{\tilde{x}} \left(J(\tilde{x}, y, \sigma_{vu}) - K(y)\right) \mathbb{1}_{\{V(\tilde{x},y,\sigma_{vu})-U(\tilde{x})>0\}} \mathbb{1}_{\{J(\tilde{x},y,\sigma_{vu})-K(y)>0\}} \mathrm{d}H_u(\tilde{x}) \\
&+ q(\theta)\frac{\xi}{s}\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{\sigma}} \left(J(\tilde{x}, y, \sigma_{ve}) - K(y)\right) \mathbb{1}_{\{V(\tilde{x},y,\sigma_{ve})-U(\tilde{x})>S(\tilde{x},\tilde{y},1)\}} \mathbb{1}_{\{J(\tilde{x},y,\sigma_{ve})-K(y)>0\}} \mathrm{d}H(\tilde{x}, \tilde{y}, \tilde{\sigma})
\end{aligned}
$$

In equilibrium, we impose a free-entry condition to determine the mass of jobs. To enter the labor market, firms pay a fixed cost c_e, and receive a productivity draw y from $G(\cdot)$. The free-entry condition is: $\int_y K(y)\mathrm{d}G(y) = c_e$

The value function of an employed worker in job (x, y, σ) is slightly more cumbersome. Employed workers receive a wage $w(x, y, \sigma)$, which is determined as an equilibrium object. They face the risk of their match being exogenously destroyed at rate η, or their firm being destroyed at rate δ, in which case they return to unemployment. They then potentially face one of eight scenarios: they meet a vacant job and move, they meet a vacant job and renegotiate à la CPVR, they meet an occupied job and move, they meet an occupied job and renegotiate, or their firm meets an unemployed or en employed worker, and they are fired and return to unemployment or are forced into a renegotiation. When fired, we assume a fraction γ of the firing costs are paid back to the worker as severance payment. The rest is simply wasted. The value function is then:

$$
\begin{aligned}
&\rho V(x,y,\sigma) = w(x,y,\sigma) + (\delta+\eta)(U(x) - V(x,y,\sigma)) \\
&+ \xi p(\theta)\frac{1}{\nu}\int_{\tilde{y}} (V(x,\tilde{y},\sigma_{ev}) - V(x,y,\sigma)) \\
&\qquad \mathbb{1}_{\{V(x,\tilde{y},\sigma_{ev})-U(x)>S(x,y,1)>V(x,y,\sigma)-U(x)\}}\mathbb{1}_{\{J(x,\tilde{y},\sigma_{ev})-K(\tilde{y})>0\}}\mathrm{d}H_v(\tilde{y}) \\
&+ \xi p(\theta)\frac{1}{\nu}\int_{\tilde{y}} (V(x,y,\sigma_{rv}) - V(x,y,\sigma)) \\
&\qquad \mathbb{1}_{\{V(x,y,\sigma_{rv})-U(x)>S(x,\tilde{y},1)>V(x,y,\sigma)-U(x)\}}\mathbb{1}_{\{J(x,y,\sigma_{rv})-K(y)>0\}}\mathrm{d}H_v(\tilde{y}) \\
&+ \xi p(\theta)\frac{\chi}{\nu}\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{\sigma}} (V(x,\tilde{y},\sigma_{eo}) - V(x,y,\sigma)) \\
&\qquad \mathbb{1}_{\{V(x,\tilde{y},\sigma_{eo})-U(x)>S(x,y,1)>V(x,y,\sigma)-U(x)\}}\mathbb{1}_{\{J(x,\tilde{y},\sigma_{eo})-K(\tilde{y})-c_f>S(\tilde{x},\tilde{y},0)>J(\tilde{x},\tilde{y},\tilde{\sigma})-K(\tilde{y})\}}\mathrm{d}H(\tilde{x},\tilde{y},\tilde{\sigma}) \\
&+ \xi p(\theta)\frac{\chi}{\nu}\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{\sigma}} (V(x,y,\sigma_{ro}) - V(x,y,\sigma)) \\
&\qquad \mathbb{1}_{\{V(x,y,\sigma_{ro})-U(x)>S(x,\tilde{y},\sigma_{reneg})-c_f-S(\tilde{x},\tilde{y},0)>V(x,y,\sigma)-U(x)\}}\mathbb{1}_{\{J(x,y,\sigma_{ro})-K(y)>0\}}\mathrm{d}H(\tilde{x},\tilde{y},\tilde{\sigma}) \\
&+ \chi q(\theta)\frac{1}{s}(U(x) + \gamma c_f - V(x,y,\sigma)) \\
&\qquad \int_{\tilde{x}} \mathbb{1}_{\{V(\tilde{x},y,\sigma_{ou})-U(\tilde{x})>0\}}\mathbb{1}_{\{J(\tilde{x},y,\sigma_{ou})-c_f-K(y)>S(x,y,0)>J(x,y,\sigma)-K(y)\}}\mathrm{d}H_u(\tilde{x}) \\
&+ \chi q(\theta)\frac{1}{s}\int_{\tilde{x}} (V(x,y,\sigma_{ru}) - V(x,y,\sigma)) \\
&\qquad \mathbb{1}_{\{V(x,y,\sigma_{ru})-U(x)>0\}}\mathbb{1}_{\{J(x,y,\sigma_{ru})-K(y)>S(\tilde{x},y,0)-c_f>J(x,y,\sigma)-K(y)\}}\mathrm{d}H_u(\tilde{x}) \\
&+ \chi q(\theta)\frac{\xi}{s}(U(x) + \gamma c_f - V(x,y,\sigma))\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{\sigma}} \\
&\qquad \mathbb{1}_{\{V(\tilde{x},y,\sigma_{oe})-U(\tilde{x})>S(\tilde{x},\tilde{y},1)>V(\tilde{x},\tilde{y},\tilde{\sigma})-U(\tilde{x})\}}\mathbb{1}_{\{J(\tilde{x},y,\sigma_{oe})-c_f-K(y)>S(x,y,0)>J(x,y,\sigma)-K(y)\}}\mathrm{d}H(\tilde{x},\tilde{y},\tilde{\sigma}) \\
&+ \chi q(\theta)\frac{\xi}{s}\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{\sigma}} (V(x,y,\sigma_{re}) - V(x,y,\sigma)) \\
&\qquad \mathbb{1}_{\{V(x,y,\sigma_{re})-U(x)>0\}}\mathbb{1}_{\{J(x,y,\sigma_{re})-K(y)>S(\tilde{x},y,\sigma_{reneg})-c_f-S(\tilde{x},\tilde{y},1)>J(x,y,\sigma)-K(y)\}}\mathrm{d}H(\tilde{x},\tilde{y},\tilde{\sigma})
\end{aligned}
$$

Finally, the value function of an occupied job (x, y, σ) is very similar to what has been described so far. A firm collects the output value of the match $z(x, y)$, and has to pay a wage $w(x, y, \sigma)$. The job can be exogenously destroyed at rate δ, in which case the continuation value is 0. The match can also be exogenously destroyed, and the firm would return to being vacant. A firm can then face one of eight scenarios described above: their worker can meet a vacant or an occupied job and move, in which case they return to being vacant, or force them to renegotiate; or they can meet an unemployed or an employed worker, and replace their own worker provided they pay firing costs, or force them to renegotiate. The value function is then:

$$
\begin{aligned}
&\rho J(x,y,\sigma) = z(x,y) - w(x,y,\sigma) + \delta(0 - J(x,y,\sigma)) + \eta(K(y) - J(x,y,\sigma)) \\
&+ \xi p(\theta)\frac{1}{\nu}\left(K(y) - J(x,y,\sigma)\right)\int_{\tilde{y}} \\
&\qquad \mathbb{1}_{\{V(x,\tilde{y},\sigma_{ev})-U(x)>S(x,y,1)>V(x,y,\sigma)-U(x)\}}\mathbb{1}_{\{J(x,\tilde{y},\sigma_{ev})-K(\tilde{y})>0\}}\mathrm{d}H_v(\tilde{y}) \\
&+ \xi p(\theta)\frac{1}{\nu}\int_{\tilde{y}}\left(J(x,y,\sigma_{rv}) - J(x,y,\sigma)\right) \\
&\qquad \mathbb{1}_{\{V(x,y,\sigma_{rv})-U(x)>S(x,\tilde{y},1)>V(x,y,\sigma)-U(x)\}}\mathbb{1}_{\{J(x,y,\sigma_{rv})-K(y)>0\}}\mathrm{d}H_v(\tilde{y}) \\
&+ \xi p(\theta)\frac{\chi}{\nu}\left(K(y) - J(x,y,\sigma)\right)\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{\sigma}} \\
&\qquad \mathbb{1}_{\{V(x,\tilde{y},\sigma_{eo})-U(x)>S(x,y,1)>V(x,y,\sigma)-U(x)\}}\mathbb{1}_{\{J(x,\tilde{y},\sigma_{eo})-K(\tilde{y})-c_f>S(\tilde{x},\tilde{y},0)>J(\tilde{x},\tilde{y},\tilde{\sigma})-K(\tilde{y})\}}\mathrm{d}H(\tilde{x},\tilde{y},\tilde{\sigma}) \\
&+ \xi p(\theta)\frac{\chi}{\nu}\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{\sigma}}\left(J(x,y,\sigma_{ro}) - J(x,y,\sigma)\right) \\
&\qquad \mathbb{1}_{\{V(x,y,\sigma_{ro})-U(x)>S(x,\tilde{y},\sigma_{reneg})-c_f-S(\tilde{x},\tilde{y},0)>V(x,y,\sigma)-U(x)\}}\mathbb{1}_{\{J(x,y,\sigma_{ro})-K(y)>0\}}\mathrm{d}H(\tilde{x},\tilde{y},\tilde{\sigma}) \\
&+ \chi q(\theta)\frac{1}{s}\int_{\tilde{x}}\left(J(\tilde{x},y,\sigma_{ou}) - c_f - J(x,y,\sigma)\right) \\
&\qquad \mathbb{1}_{\{V(\tilde{x},y,\sigma_{ou})-U(\tilde{x})>0\}}\mathbb{1}_{\{J(\tilde{x},y,\sigma_{ou})-c_f-K(y)>S(x,y,0)>J(x,y,\sigma)-K(y)\}}\mathrm{d}H_u(\tilde{x}) \\
&+ \chi q(\theta)\frac{1}{s}\int_{\tilde{x}}\left(J(x,y,\sigma_{ru}) - J(x,y,\sigma)\right) \\
&\qquad \mathbb{1}_{\{V(x,y,\sigma_{ru})-U(x)>0\}}\mathbb{1}_{\{J(x,y,\sigma_{ru})-K(y)>S(\tilde{x},y,0)-c_f>J(x,y,\sigma)-K(y)\}}\mathrm{d}H_u(\tilde{x}) \\
&+ \chi q(\theta)\frac{\xi}{s}\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{\sigma}}\left(J(\tilde{x},y,\sigma_{oe}) - c_f - J(x,y,\sigma)\right) \\
&\qquad \mathbb{1}_{\{V(\tilde{x},y,\sigma_{oe})-U(\tilde{x})>S(\tilde{x},\tilde{y},1)>V(\tilde{x},\tilde{y},\tilde{\sigma})-U(\tilde{x})\}}\mathbb{1}_{\{J(\tilde{x},y,\sigma_{oe})-c_f-K(y)>S(x,y,0)>J(x,y,\sigma)-K(y)\}}\mathrm{d}H(\tilde{x},\tilde{y},\tilde{\sigma}) \\
&+ \chi q(\theta)\frac{\xi}{s}\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{\sigma}}\left(J(x,y,\sigma_{re}) - J(x,y,\sigma)\right) \\
&\qquad \mathbb{1}_{\{V(x,y,\sigma_{re})-U(x)>0\}}\mathbb{1}_{\{J(x,y,\sigma_{re})-K(y)>S(\tilde{x},y,\sigma_{reneg})-c_f-S(\tilde{x},\tilde{y},1)>J(x,y,\sigma)-K(y)\}}\mathrm{d}H(\tilde{x},\tilde{y},\tilde{\sigma})
\end{aligned}
$$

3.3 Surplus

Recall that we define the surplus of a match (x,y,σ) by $S(x,y,\sigma) = V(x,y,\sigma) - U(x) + J(x,y,\sigma) - V(y)$. Summing up the previous value functions, using the bargaining and the transition rules, and finally noting that S does not depend on σ, we can show that S follows:

$$
\begin{aligned}
&\rho S(x,y) = z(x,y) - b + c_v - (\delta+\eta)S(x,y) \\
&+ \xi p(\theta)\frac{1}{\nu}\int_{\tilde{y}} \alpha\left(S(x,\tilde{y}) - S(x,y)\right)\mathbb{1}_{\{S(x,\tilde{y})>S(x,y)\}}\mathrm{d}H_v(\tilde{y}) \\
&+ \xi p(\theta)\frac{\chi}{\nu}\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{\sigma}} \alpha\left(S(x,\tilde{y}) - S(x,y) - S(\tilde{x},\tilde{y}) - c_f\right)\mathbb{1}_{\{S(x,\tilde{y})>S(x,y)+c_f+S(\tilde{x},\tilde{y})\}}\mathrm{d}H(\tilde{x},\tilde{y},\tilde{\sigma}) \\
&+ \chi q(\theta)\frac{1}{s}\int_{\tilde{x}} \left((1-\alpha)\left(S(\tilde{x},y) - S(x,y)\right) + (\gamma-(1-\alpha))c_f\right)\mathbb{1}_{\{S(\tilde{x},y)>c_f+S(x,y)\}}\mathrm{d}H_u(\tilde{x}) \\
&+ \chi q(\theta)\frac{\xi}{s}\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{\sigma}} \left((1-\alpha)\left(S(\tilde{x},y) - S(\tilde{x},\tilde{y}) - S(x,y)\right) + (\gamma-(1-\alpha))c_f\right) \\
&\qquad \mathbb{1}_{\{S(\tilde{x},y)>c_f+S(x,y)+S(\tilde{x},\tilde{y})\}}\mathrm{d}H(\tilde{x},\tilde{y},\tilde{\sigma}) \\
&- p(\theta)\frac{1}{\nu}\int_{\tilde{y}} \alpha S(x,\tilde{y})\mathbb{1}_{\{S(x,\tilde{y})>0\}}\mathrm{d}H_v(\tilde{y}) \\
&- p(\theta)\frac{\chi}{\nu}\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{\sigma}} \alpha\left(S(x,\tilde{y}) - c_f - S(\tilde{x},\tilde{y})\right)\mathbb{1}_{\{S(x,\tilde{y})>c_f+S(\tilde{x},\tilde{y})\}}\mathrm{d}H(\tilde{x},\tilde{y},\tilde{\sigma}) \\
&- q(\theta)\frac{1}{s}\int_{\tilde{x}}(1-\alpha)S(\tilde{x},y)\mathbb{1}_{\{S(\tilde{x},y)>0\}}\mathrm{d}H_u(\tilde{x}) \\
&- q(\theta)\frac{\xi}{s}\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{\sigma}}(1-\alpha)(S(\tilde{x},y) - S(\tilde{x},\tilde{y}))\mathbb{1}_{\{S(\tilde{x},y)>S(\tilde{x},\tilde{y})\}}\mathrm{d}H(\tilde{x},\tilde{y},\tilde{\sigma})
\end{aligned}
$$

We verify in appendix A that when we increase the costs of firing, *de facto* preventing firing, we retrieve the usual CPVR surplus function.

4 Towards an efficient level of firing costs

4.1 Numerical solution, functional forms and parameters

We now solve for the surplus equation following a numerical method very close to Achdou et al. (2022). Contrary to more standard search-and-matching methods for which we can apply Achdou et al. out-of-the-shelf, we face two main issues. First, the surplus equation is not increasing in y given x, nor vice-versa, as is made clear when we plot the surplus below. If we take the usual "worker-centered" perspective in which $S(x,y) = V(x,y,\sigma) + J(x,y,\sigma) - U(x)$, S is increasing in y for a given x. This allows us to restrict our attention to increasing functions in y for each x thanks to the contraction mapping theorem. In the case we study, a bad worker has a surplus decreasing in the firm's type. The explanation is intuitive - a good firm matched with a bad worker has low output, a lower search efficiency, and would have to pay firing costs to replace its worker. The value function of a vacant firm $K(y)$ therefore increases faster than the private value of the match $V(x, y, \sigma) + J(x, y, \sigma)$. S is decreasing or hump-shaped in y given x for a number of x values. We cannot look for an increasing function in y in our value-function iteration, rendering the problem less stable numerically. Based on numerical simulation (some of which are drawn below), it seems that the private value of the match, $\Omega(x, y) = V(x, y, \sigma) + J(x, y, \sigma)$, and the value functions of an unemployed worker and of a vacant job $U(x)$, $V(y)$, are increasing in their respective argument, keeping the other one fixed. Solving our surplus equation in terms of the private value is therefore more stable.

Second, we cannot use the "conservation of mass" to find the stationary distribution of mass, by simply taking the adjunct operator of the surplus transition matrix. Intuitively, the transition matrix relies on keeping track of the mass following transitions in the labor market. Here though, a unit of match is made of a unit of worker and a unit of firm. It is as if, when we create a match between an unemployed worker and a vacant job, we take a unit mass of each, to create a single unit mass of a match. When computing the stationary distribution from the transition matrix, we cannot use the surplus (or private value) transition matrix. Instead, we need to write the transition matrix of workers and firms separately, and find its stationary distribution. We also need to ensure the conversation of mass type of worker by type of worker, and type of firm by type of firm. In standard search-and-matching, as firms only clear the market through the free entry-condition, keeping track of the mass of workers matched and unemployed is enough. Adapting Achdou et al. hence requires some work.

We then need to specify two functional forms: the matching function is Cobb-Douglas $p(\theta) = A\theta^{\beta}$ and $q(\theta) = A\theta^{-(1-\beta)}$, and the firing costs are linear in the surplus of the match, $c_f(x, y) = \mu \cdot S(x, y)$. We vary μ to model a change in the firing costs. We use standard parameter values, as described in table 1, when solving our model. Estimating the model will be challenging, as the data counter-part for several parameters is not straightforward to find. We can observe job-to-job

Parameter		Value
ρ	Discount rate	0.004
δ	Job's destruction rate	0
η	Match's destruction rate	0.005
ξ	Search efficiency of emp. workers	0.14
χ	Search efficiency of occ. job	0.07
α	Worker's bargaining power	0.3
b	Unemployment benefits	0.2
c_v	Cost of running empty job	0.1
x, y	Type of worker, firm	normal dist.
A	Matching efficiency	0.15
β	Elasticity of matching function	0.5
γ	Fraction of firing costs rebated to worker	0

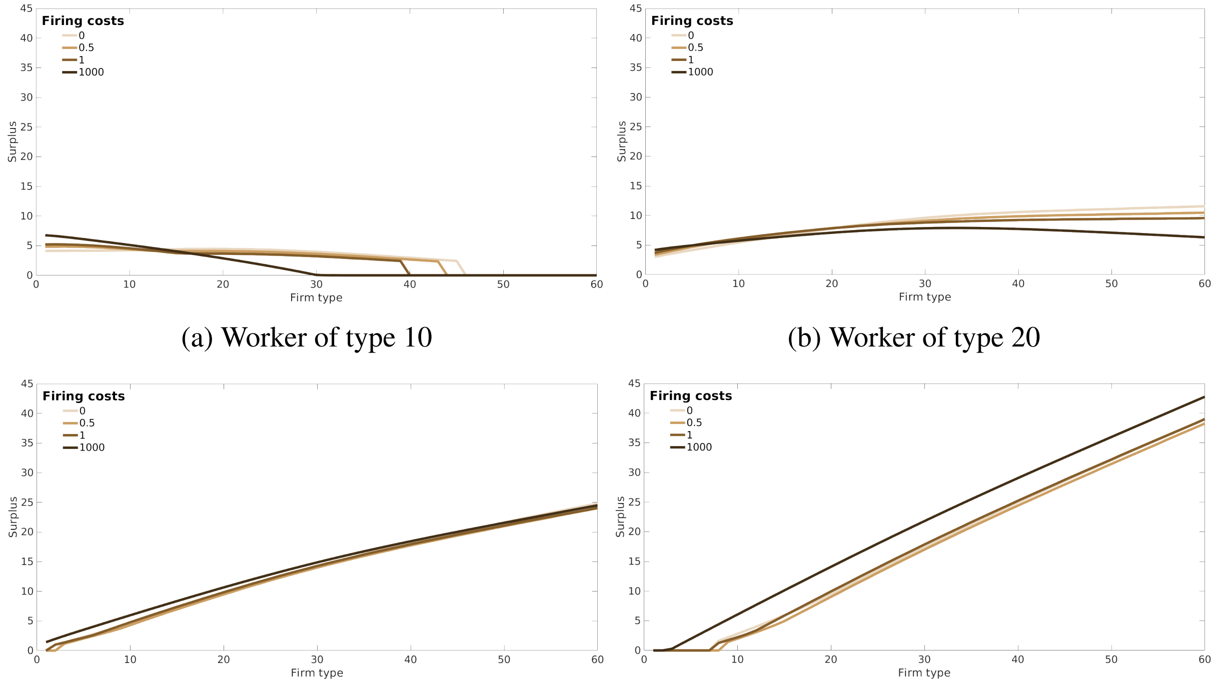
Table 1: Parameters used in numerical simulations

transitions in the data. Yet, how to isolate "worker-to-worker" transitions, in which firms replace their workers on the same job, is more complicated to isolate. What to consider an empty vacancy is also not self-explanatory in the data. We chose reasonable values for those parameters, and leave the task of estimating them to future work. We focus instead on the efficiency of firing costs and their distributional impacts given those parameters.

4.2 Efficiency, inequalities and firing costs

We solve for the equilibrium surplus and match distribution. We draw workers' and firms' types from a discretized normal distribution with 50 different types for workers and 60 for firms, with type 1 being the lowest type for both. We turn off firm's entry and exit to understand the role of firing costs in a static environment. As firms are infinitely lived, they can wait for a better match longer. In the extreme case where firms live for a very short period of time, firing costs do not matter as firms do not have time to rematch anyways. Our results should be seen as an upper-bound on the effect of firing costs from that perspective.

Figure 1 displays the effect of firing costs on the surplus for different matches. We select 4 types of workers, types 10, 20, 30, and 40, and look at how their surplus with the different firm's types evolve when we vary the firing costs from 0 to a prohibitively high value. Firing costs



(a) Worker of type 10

(b) Worker of type 20

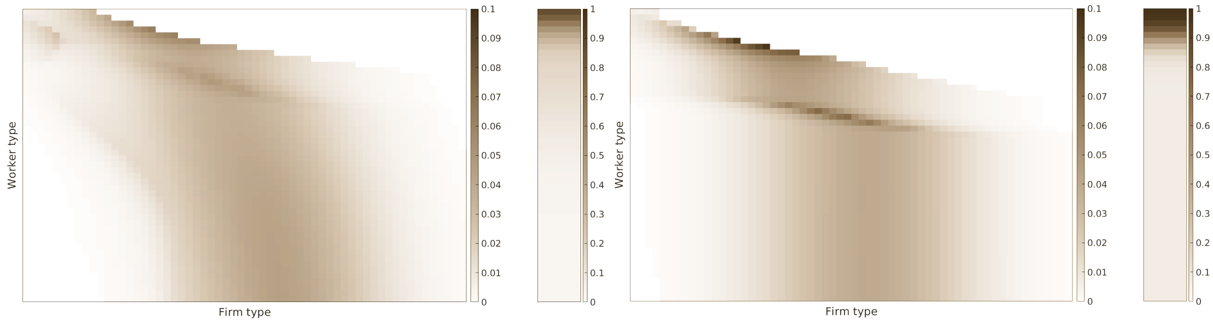
(c) Worker of type 30

(d) Worker of type 40

Figure 1: Evolution of the surplus of a match for different firing costs

have a highly non-linear effect. They reduce the value of the surplus for low-type workers, and increase the value for high-type workers. Understanding why the surplus for high-type workers increase is relatively straightforward. High-type workers are on average never replaced. Given that the productivity are normally distributed, and that low-type workers tend to be unemployed much more frequently, meeting a high-type worker is a rare event. Meeting a higher-type worker worth replacing her by, especially if one needs to pay firing costs in addition, is an extremely rare event. High-type workers do not need firing costs to protect them from displacement. What firing costs do though, is to reduce the value of being unemployed. A high-type unemployed worker has a high chance of replacing an incumbent in case of a meeting with an occupied job. The higher the firing costs, the least opportunities an unemployed worker would have to do so, decreasing the value of unemployment, and therefore increasing the surplus of a match.

For low-type workers, the effect is more nuanced. It increases the value of the surplus with low-type firms, but decreases it with high-type firms. When firms cannot fire, the value of searching for a firm when the job is occupied decreases. A good firm prefers to wait to meet a better worker,



(a) No firing costs (b) Prohibitive firing costs

Figure 2: Distribution of workers across matches and unemployment by types for different firing costs

rather than having to remain with a bad match, decreasing the value of a match with a higher-type firm. With low-type firms, higher firing costs stabilize a match by preventing firms from replacing their workers, increasing the value of the match. Looking at the surplus only, it seems that, if anything, firing costs help to stabilize bad matches, which helps low-type workers.

Focusing on the distribution of workers across matches paints a bleaker picture. Figure 2 plots the mass of each type of workers across the different firms and unemployment, for two different firing costs - no firing costs, and prohibitive firing costs. Each row represents a type of worker, and sums up to a mass of 1 when one adds the mass across firm types and unemployment (last column, to the right of each matrix). The darker the heatmap, the more workers of a given type are in that state. Workers of the lowest type are never matched with good firms. In fact, very few of them are matched at all, and most are in the last column, in unemployment. High-type workers are matched across a wide range of middle to high type firms, and very few are unemployed. Recall search is random, and the distribution of firm's types is normal - explaining why, on average, there are very few high-high matches. Overall our model depicts some sort of assortative matching at the aggregate level, as could be expected with a multiplicative production function and heterogeneity on both sides of the market.

Comparing both panels, the average matching heatmap seems more assortative with high firing costs. Low-type workers tend to remain unemployed even more than when firing costs are low. Firms do not want to hire and remain in a bad match until it exogenously destroys. They instead

Value of μ (firing costs: $c_f(x,y) = \mu S(x,y)$)	**0**	**0.25**	**0.5**	**0.75**	**1**
Total output	0.9525	0.9522	0.9512	0.9501	0.9493
% increase in output compared to no firing	**0.77**	**0.74**	**0.63**	**0.52**	**0.43**
Unemployment rate (%)	7.23	7.18	7.19	7.22	7.25
Value of μ (firing costs: $c_f(x,y) = \mu S(x,y)$)	**2**	**5**	**10**	**100**	**1000**
Total output	0.9474	0.946	0.9456	0.9452	0.9452
% increase in output compared to no firing	**0.23**	**0.08**	**0.04**	**0**	**0**
Unemployment rate (%)	7.35	7.41	7.42	7.44	7.43

Table 2: Unemployment rate and total output by firing costs

decide not to hire bad workers. Even though the surplus of a bad match might be higher with high firing costs, low-type workers are much worse off when firms cannot search while the job is occupied. If firing costs are designed to protect low-type workers who would struggle to find another job, it seems that they are doing the exact opposite. By preventing firms from replacing their workers, they do not hire them in the first place.

In terms of efficiency, firing costs reduce total output and increase unemployment, as shown in table 2, although the effects do not appear to be large in our simulations. One would gain less than 1% of total output and reduce unemployment by 0.2% by going from no firing to free firing. Given that many elements are not modeled here, including firm's entry and exit, or severance payments to workers, these numbers are likely to change. With aggregate effects that small, we can expect them to differ substantially in magnitude, so one should be cautious in taking these results at face-value.

5 Conclusion

Firing costs are often argued in favor of protecting the most vulnerable workers in the labor market. We saw in our model how firing costs can actually nudge firm into not hiring low-type workers rather than being unable to replace them. In this simple two-sided limited commitment setting, allowing for firms to actively search for better matches, increase total output and reduces unemployment, especially among low-type workers. Our results are consistent with many other theoretical works arguing that firing costs might have the exact opposite effect to the one they were designed for.

Our results remain preliminary, and we hope to include more bells and whistles to our model. This could potentially change the results. We have not yet looked at the impact on workers' wages. Through active searching, firms are able to renegotiate wages downwards. If a social planner is interested not only in output and unemployment, but also on the wage distribution, then allowing for renegotiation by lowering firing costs reduces the labor share. In what was presented above, firing costs firms have to pay are wasted, lowering output each time a worker is replaced. Severance payment could be introduced, and would mitigate the loss of output when increasing firing costs. It would also increase welfare of displaced workers, which could be important from a planner's perspective. Postel-Vinay and Turon (2014) present a model based on this intuition.

A second natural extension of our framework would be to introduce productivity shocks, either at the match level or at the aggregate level. As discussed already above, Rudanko (2009) proposes a model in which firing costs weight on the decision of firms to hire depending on the business cycle. Very related, Huckfeldt (2022) documents that low-type workers are more affected by recessions. High firing costs are likely to exacerbate this effect. Having counter-cyclical firing costs on the contrary, might nudge firms into hiring in booms and not firing in recessions, protecting lower-type workers.

Many over aspects of firing costs would be worth mentioning, although they would be far outside the scope of this project. Lise (2013) and Chaumont and Shi (2022) develop models of on-the-job search with precautionary savings, in which workers can save in a risk-free asset, partially insuring them against job-loss. The interactions between firing costs and precautionary savings for insurance purposes against a job displacement are likely important. As documented in Braxton, Herkenhoff, and Phillips (2020), unemployed workers largely maintain access to borrowing, potentially alleviating some of the risks of job displacement, and reducing the burden of low firing costs on low-type workers. Our product market is also very simplistic, as often in search-and-matching frameworks. Grinza and Quatraro (2019) document how innovation is hindered by workers' replacement by destroying human capital within firms. Output gains from reducing firing costs are so small in our exercise, that any externality firms do not account for could easily reverse them.

Finally, our model assumes perfect information when matching with a worker. Extending our model of bargaining with asymmetric information would be of first-order interest. It has not been done to our knowledge, even in much simpler cases. Giving firms a trial period to sort out the lemons in the labor market when worker's productivity is not observed, would provide a rational for low firing costs - at least at the beginning of a work relationship. We leave these different extensions for future work.

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A Surplus equation when firing costs are prohibitively high

As a sanity check, we verify that if $c_f \to \infty$, then S becomes:

$$
\begin{aligned}
&\rho S(x,y) = z(x,y) - b + c_v - (\eta+\delta)S(x,y) \\
&+ \xi p(\theta)\frac{1}{\nu}\int_{\tilde{y}} \alpha\left(S(x,\tilde{y}) - S(x,y)\right)\mathbb{1}_{\{S(x,\tilde{y})>S(x,y)\}}\mathrm{d}H_v(\tilde{y}) \\
&- p(\theta)\frac{1}{\nu}\int_{\tilde{y}} \alpha S(x,\tilde{y})\mathbb{1}_{\{S(x,\tilde{y})>0\}}\mathrm{d}H_v(\tilde{y}) \\
&- q(\theta)\frac{1}{s}\int_{\tilde{x}} (1-\alpha)S(\tilde{x},y)\mathbb{1}_{\{S(\tilde{x},y)>0\}}\mathrm{d}H_u(\tilde{x}) \\
&- q(\theta)\frac{\xi}{s}\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{\sigma}} (1-\alpha)(S(\tilde{x},y) - S(\tilde{x},\tilde{y}))\mathbb{1}_{\{S(\tilde{x},y)>S(\tilde{x},\tilde{y})\}}\mathrm{d}H(\tilde{x},\tilde{y},\tilde{\sigma})
\end{aligned}
$$

which is very close to the surplus in CPVR. Two differences arise, both coming directly from $K(y)$: we set up a positive cost of maintaining an empty vacancy c_v, and now the outside option destroyed when creating a match must account for the fact that a vacant job gives up on waiting for another unemployed or employed worker, which adds the last two terms. Using a usual free entry-condition defined type-by-type, $K(y) = c_e$, would collapse the surplus into:

$$
\begin{aligned}
&\rho S(x,y,\sigma) = z(x,y) - b - (\eta+\delta)S(x,y,\sigma) \\
&+ \xi p(\theta)\frac{1}{\nu}\int_{\tilde{y}} \alpha\left(S(x,\tilde{y}) - S(x,y)\right)\mathbb{1}_{\{S(x,\tilde{y})>S(x,y)\}}\mathrm{d}H_v(\tilde{y}) \\
&- p(\theta)\frac{1}{\nu}\int_{\tilde{y}} \alpha S(x,\tilde{y})\mathbb{1}_{\{S(x,\tilde{y})>0\}}\mathrm{d}H_v(\tilde{y}) \\
&- c_e
\end{aligned}
$$

which simply shift the surplus of a match compared to the standard CPVR.